DYNAMICS OF STRUCTURES WITH MATLAB® APPLICATIONS

ASHOK K. JAIN

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Dedicated to

my grandchildren, Dhruv and Samyukta, who made me
understand energy dissipation mechanisms
in live *dynamic systems*

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Brief Contents

Contents

Preface

It is needless to reemphasize the significance and importance of the subject of dynamics of structures when the architects and engineers are constantly craving for lightweight but strong materials, and lean and slim structures. Among the whole family of the subjects on *Structural Engineering*, this is indeed the most mathematical and, therefore, most scary. There are an umpteen number of textbooks available on the subject of Dynamics of Structures that discuss the derivation of equations along with their physical significance. Nevertheless, it is very difficult to visualize these equations. FORTRAN has been the most powerful and preferred programming language of structural engineers, but it was without graphic commands. The subjects of structural dynamics, stiffness matrix method, nonlinear behaviour of members and structures, FORTRAN language, and computer hardware and software have evolved together since early 1960s. There used to be special Tektronix and vt100 graphic terminals, and Calcomp plotters with proprietary graphics software. The biggest hurdle in understanding the subject of dynamics remained the inability of easy graphical representation of equations and dynamic response of structures. Remember SAP IV, DRAIN-2D and SAKE developed in early 1970s? With the beginning of the twenty-first century, all that has changed. The desktop and laptop computers, and colour laser printers with very high resolutions are easily available and affordable. MATLAB is a very powerful and user-friendly software for carrying out solution of extremely complicated mathematical equations with built-in graphic functions. Another very easy and powerful tool is the electronic worksheet such as MS-EXCEL.

The writing of this book was inspired by the following objectives:

- to present the subject matter with utmost ease,
- to provide necessary and detailed mathematical background,
- to introduce more illustrative examples,
- to present the subject matter useful to final year undergraduate and postgraduate students,
- to introduce MS-EXCEL and MATLAB,
- to introduce nonlinear modelling and analysis,
- to introduce special damping devices and their modelling, and
- to introduce practical applications useful to practicing engineers.

During the past few years, my students have greatly appreciated the power and impact of these tools in understanding the subject. An attempt has been made to present both elementary topics as well as advanced topics including acceleration–displacement response spectra (ADRS) and performance-based seismic design of structures. The response of structures with energy dissipating devices subjected to earthquakes is also presented.

ORGANIZATION OF THE BOOK

A genuine effort has been made to develop the subject from the very basics of simple harmonic motion of a pendulum and introduce the concept of equation of motion. The next step is to find its solution. There are several techniques to solve the equations of motion for different types of dynamic loads. The analysis of structures due to different dynamic loads has been carried out in Chapters 3 to 7. The estimation of earthquake force has been discussed in Chapter 8. Analysis of two degrees of freedom system and tuned mass dampers has been developed in Chapter 9, whereas that of multi-degree-of-freedom systems has been developed in Chapters 10 and 11.

The analysis of multistorey reinforced concrete and steel buildings subjected to earthquake loads in accordance with IS:1893 code has been discussed in Chapter 12. Under a severe earthquake loading, a structure is expected to undergo inelastic region. Modelling for nonlinear analysis, hysteresis models, solution algorithms, energy dissipating devices, concept of ductility etc. are discussed in Chapter 13. Nowadays, there is a great emphasis on predicting the performance of a structure under earthquake loads. The intention is to know whether the structure will remain in *immediate occupancy, damage control, life safety, limited safety, or in structural stability states* during an earthquake event. In case there is a downtime for the building after an event, then how much will it be? What it will cost to the owner and its occupants as a result of downtime? What will be the estimated extent and cost of repair? Pushover analysis is used to study the performance-based design. These issues are discussed in Chapter 14. As on today, we may not have all the answers but these do indicate the direction of further research.

Finally, the last Chapter 15 is devoted to the estimation of wind loads based on IS:875-Part 3 and IRC6. Wind may be treated as a static load or dynamic load. The wind loads on various structures are calculated based on exhaustive studies in wind tunnels over an extended period of time in various countries. The concepts of fluid mechanics are involved in the estimation of wind loads. It is important to understand the estimation of drag coefficient for different shape and size of structures and their exposed structural elements. It is interesting to know that the dynamic wind loads are applied statically to a structure to understand its response.

Wherever required, IS:1893, IS:875-part 3, IS:2974, IRC 6, Eurocode 8, ASCE 7, AISC 341, NZS 1170 and ISO codes have been introduced. Federal Emergency Management Agency (FEMA), Washington, D.C., and Pacific Earthquake Engineering Research Center (PEER) have prepared several documents with detailed commentary and background notes including publications under the National Earthquake

Hazards Reduction Program (NEHRP).These publications have been introduced as appropriate. It is recommended that the reader should have a copy of these codes and research reports to understand and appreciate the latest developments.

HOW TO STUDY THE SUBJECT OF DYNAMICS?

As already pointed out, this is a highly mathematical subject. It is recommended that the reader should himself/herself derive each equation and make it a general practice to represent them in graphical form. It will help develop an understanding of the nature of equations, their physical meaning and interpretation, and, therefore, behaviour of the structure under a given dynamic load. MATLAB is a very powerful tool for learning and exploring the subject. MS-EXCEL is another very powerful tool to carry out repetitive calculations and represent the data in graphical form. In addition, the dynamic response of structures subjected to earthquake loading should be understood using commercially available software such as SAP2000 and ETABS. The GUI in all these tools is extremely powerful and helpful in unravelling the mystery of dynamics of structures. All such tools must be fully exploited for effective computer-aided learning.

MATLAB source codes developed in this text can be obtained by requesting at www.pearsoned.co.in/ashokkjain

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I wish to thank the following students with whom I had short but intense brainstorming sessions, *sometimes forced as is natural in dynamics*, on various aspects of structural dynamics while they were working on their research projects during their stay at Roorkee: M. R. Deshpande, S. S. Dasaka, Shri Pal, R. A. Mir, Satish Annigeri, Jainendra Agarwal, J. Satyanarayan, P. Rajeshwari, M. L. Meena, Abhijit S. Niphade, Sourabh Agrawal, Ranjith Shetty, Sujit Ashok Gangal, Payal Thukral, Abhinav Gupta, Shabbir Lokhandwala, Mandakini Dehuri, Pruthvik B. M., Alwin N., Ripu Daman Singh and Saurabh Khandelwal. In addition, there were several other students who worked on static problems and had very stimulating technical sessions with me.

Special thanks to Ashok Mittal for our long-distance telephonic discussions on various aspects of earthquake engineering from the point of view of a structural designer who was chasing deadlines to finalize computer models and issue structural drawings, and there remained a few fundamental issues still unresolved. I must admit that I learnt wind loads from A. K. M. Tripathi who had a deep understanding of wind loads on TV and MW towers. Sincere thanks to Aparna K. P. who worked with me for her Master's thesis with a clean slate and in a very short time picked up the fine points of inelastic response spectra and acceleration–displacement response spectra, and helped produce numerous tables and graphs.

How can I forget my *alma mater*, the erstwhile University of Roorkee and now Indian Institute of Technology Roorkee, situated in a very small and calm town on the banks of Ganga and foothills of Himalaya, for providing an excellent work environment, library and computing facilities to learn, learn and learn? The University of Michigan at Ann Arbor, my another *alma mater*, provided me with excellent laboratory facilities in its North Campus to generate hysteresis loops for steel bracing members, and computing facilities in its main campus to study the inelastic seismic response of concentrically and eccentrically braced steel frames.

I acknowledge the editorial and production teams at Pearson consisting of R. Dheepika, C. Purushothaman and Sojan Jose for their untiring efforts and tolerating my last-minute changes in producing this book in the present form.

Lastly, I wish to thank my wife Sarita, our children Payal and Gaurav, son-in-law Vikash and daughter-in-law Saavy, for their unconditional support and encouragement in writing this book. I also thank my father who constantly advised me to write a book exclusively on earthquake engineering but some how I wasn't convinced.

Ashok K. Jain

About the Author

Dr Ashok K. Jain is Professor of Civil Engineering at the Indian Institute of Technology Roorkee (formerly University of Roorkee), obtained his B.E. and M.E. degrees with honours from the University of Roorkee in 1972 and 1974, and a doctorate degree from the University of Michigan, Ann Arbor, in 1978. His main areas of interest include multistoreyed buildings, concrete and steel bridges, and nonlinear seismic response of structures. Besides teaching and research, he has been a structural consultant to various state and central government agencies as well as many private companies. A recipient of several awards, he has been a research fellow at the University of Michigan; a visiting Professor at the McGill University, Montreal; Director, Malaviya National Institute of Technology, Jaipur; and Head of Civil Engineering Department, I.I.T. Roorkee.

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PART

1

SINGLE DEGREE OF FREEDOM SYSTEMS

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1 Introduction to Structural Dynamics

1.1 INTRODUCTION

There are many situations in real life when a structure is subjected to vibrations caused by *dynamic loads* due to machines, road traffic, rail traffic or air traffic, wind, earthquake, blast loading, sea waves, or tsunami. The movement of pedestrians may cause vibrations in a floor of a building and in a suspension bridge. The term *dynamic loads* includes any loading which varies with time. The manner in which a *structure* responds to a given dynamic excitation depends upon the nature of excitation and the dynamic characteristics of the structure, that is, the manner in which it stores and dissipates energy. The energy is stored in the form of potential energy and is dissipated in the form of kinetic energy through vibrations. Some of these loads could lead to catastrophic loss to lives and properties in a very short span of time while the others cause irritation to the users of the facility. If the vibrations are small but persistent, this condition may lead to cracking in joints and members of a structure. In earlier days, a simplified solution was arrived at by treating the *dynamic load* as *equivalent static load*. Under certain conditions, this idealization works quite well. However, with better understanding of dynamics of structures, it is desirable to develop more efficient solutions by realistically modelling the loads taking account of their dynamic nature. When a dynamic load is applied to the structure it produces a time-dependent response in each element of the structure. The organization of structural systems to resist such loadings has a major influence on the overall planning, design and economics of a structure.

The subject of dynamics of structures becomes quite complex because of a large number of variables involved in the problem. The term *structure* encompasses a single-storey building to a multi-storey building, small overhead water tanks to large water tanks of different shapes, small bridges to large bridges of different materials and configurations, viaducts, microwave and TV towers, industrial structures, machine foundations, theatres, stadiums, airports, jetties and so on. Mass and stiffness of a structure along with their spatial distribution define the structure. Material used in structures is another parameter that defines the structure. Damping is generally associated with materials. Finally the *amplitude, frequency* and *duration of load* lead to another complication. Earthquake loading is a complex variant of this. Some of the dynamic loads are *deterministic* while the others are *non-deterministic or probabilistic*. Some of the structures will respond to dynamic loads in *linear and elastic* range while the others may venture into *nonlinear elastic* or *nonlinear inelastic* range. The role of a structural analyst is to identify each of these parameters and determine the response of the structure as accurately as possible. The study of vibrations, that is, their cause, measurement, analysis and effect on the structure is called *dynamics of structures*.

The details of excitation, structure and response are illustrated in Figures 1.1 to 1.5. The excitation shown in Figures 1.2(b) and 1.2(c) may be static or dynamic or even slow dynamic. A slow dynamic load is also called pseudo static load. It is very helpful in conducting tests in laboratory.

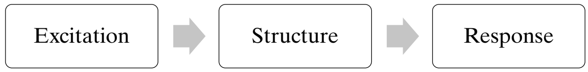


Figure 1.1 Cause and effect on a structure.

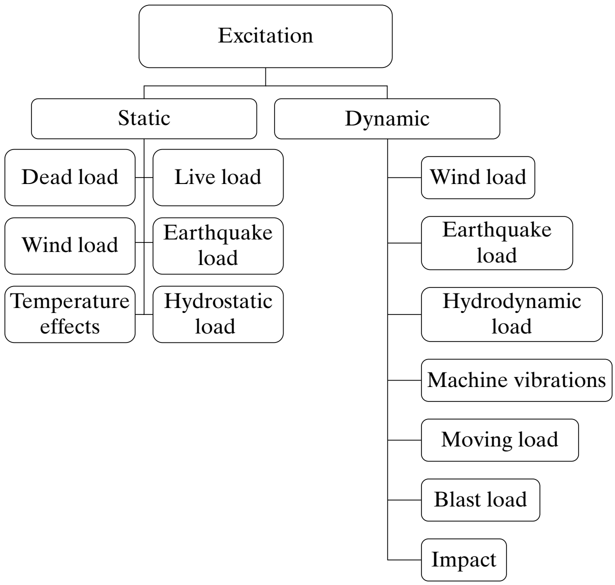


Figure 1.2(a) Source of excitation.

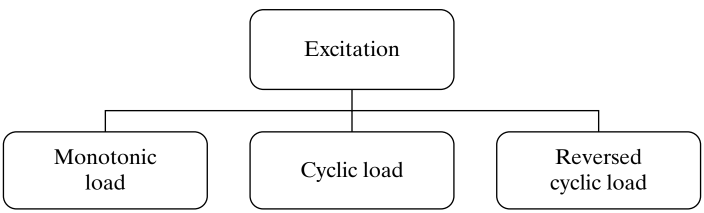
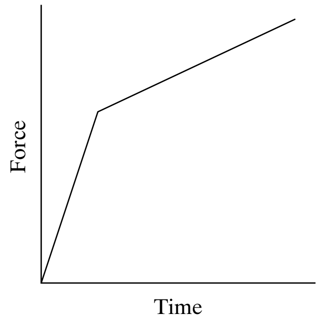
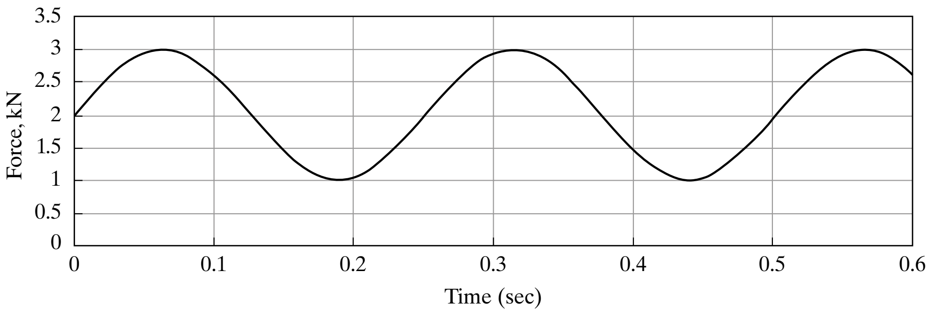


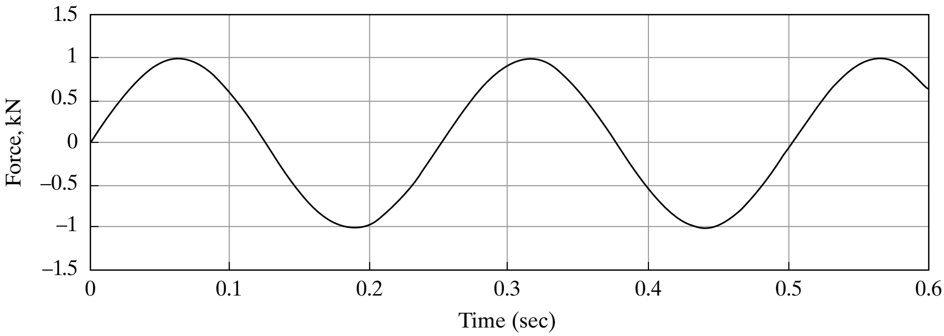
Figure 1.2(b) Nature of excitation.



(i) Monotonic loading



(ii) Cyclic loading



(iii) Reversed cyclic loading

Figure 1.2(c) Nature of excitation.

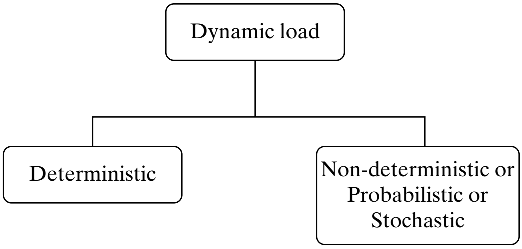


Figure 1.3 Assessment of dynamic load.

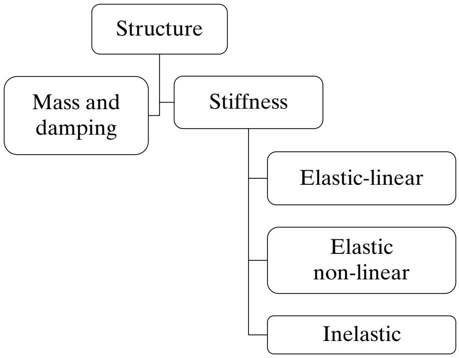


Figure 1.4 Properties of a structure.

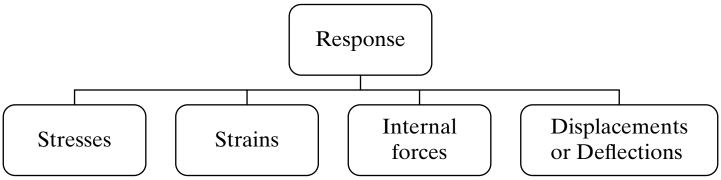


Figure 1.5 Type of response.

1.1.1 Why Dynamic Analysis?

The objective of dynamic analysis is to determine the displacement–time history, velocity–time history and acceleration–time history, and internal member force – time history due to the given force–time history. The time history provides the following information:

- Peak values (positive and negative)
- Mean value
- Nature of change in the response parameter—that is, its frequency (only qualitatively)
- Oscillations with respect to its mean position.

It is possible to determine absolute maximum displacement, absolute maximum velocity and absolute maximum acceleration. Knowing stiffness of the structure, forces in various members can be calculated. The dynamic member forces can be superimposed on those due to static loads in different load combinations. The design member forces can be estimated for the chosen design philosophy, that is, *limit state design* or *performance based design philosophy*.

Let us discuss some of the terminologies used in dynamic analysis of structures.

Amplitude The maximum value of a periodic function (which may be displacement, velocity, acceleration or force) is called amplitude.

Damping It is the dissipation of energy that causes the amplitude of motion of a freely vibrating structure to decrease with each cycle with the motion eventually dying out. The term 'damping' is widely applied to energy dissipation mechanisms that are not associated with structural damage. It may be natural or artificial, that is, external. Natural damping is an inherent property of the material. There are different types of damping such as viscous damping, friction damping and hysteretic damping.

Degree of Freedom The number of independent displacement components required to define the deflected shape of a structure is called degree of freedom. If only a single displacement is required to define its deflected shape, it is called a single-degree-of-freedom (SDOF) system. If 'n' degrees are required, it is called a multi-degrees-of-freedom system.

Deterministic When a given parameter is fully known and there are no uncertainties surrounding the values of the parameter, it is called deterministic.

Forced Vibration When a structure vibrates under the influence of an external force, it is called forced vibration.

Free Vibration When a structure vibrates without any externally applied force such as when it is pulled out of position and then released gently, it is called free vibration.

Frequency The number of cycles of motion completed during a unit time interval (i.e., radian per second or cycles per second) is called frequency. Frequency is the inverse of period (that is, a structure with a natural period of 0.5 second has a natural frequency of 2 cycles per second or 2 Hz). When a structure vibrates freely without any external force, it is said to have one or more *natural frequency* or *natural period* of vibration depending upon its degrees of freedom.

Fundamental Frequency The smallest frequency of vibration of a structure is called the fundamental frequency.

Fundamental Period The longest period of vibration of a structure is called the fundamental period.

Mode Shape A structure is assumed to vibrate in different modes depending upon its natural frequencies. The total number of modes depends upon the total active degrees of freedom of the structure. There will be a unique mode of vibration corresponding to each natural frequency.

Non-deterministic or Probabilistic When a given parameter is not fully known and there are uncertainties surrounding the values of the parameter, it is called non-deterministic or probabilistic. Thus, probability of uncertainties associated with all variables used explicitly or implicitly to define the parameter would be formally taken into account.

Period The duration of one cycle of motion in second is called period.

Resonance The amplification of response that occurs under forced vibration when period of the applied force is equal or very close to the natural period of the structure is called resonance. The amplitude of vibrations is maximum at resonance and may lead to damage to the machine, structure or its foundation.

Steady State Vibration When a structure vibrates under the influence of an external periodic force, it is said to be in a steady state.

Stochastic It is synonymous with the term 'random'. It is of Greek origin and means pertaining to chance or randomly determined sequence of observations. It is same as *non-deterministic*.

Transient Vibration These vibrations depend upon the initial conditions of the structure (displacement and velocity at time $t = 0$) and die out shortly due to the inherent damping present in a structure.

1.2 PHYSICAL AND MATHEMATICAL MODELLING

Civil engineering structures are conceived to meet the functional and aesthetical requirements. They consist of a combination of structural elements and non-structural elements. It is a standard practice to idealize a physical problem into a mathematical problem in order to study its response under the applied loads. It may be modelled as a one-, two- or a three-dimensional problem as shown in Figure 1.6. The non-structural elements are generally not modelled. Next, it is necessary to determine the *force-deformation* relationship for this mathematical model. For small deformations, the force-deformation relation is usually *linear and elastic*. The slope of this relation gives stiffness of the structure. At larger displacements, the slope may become *non-linear*. The structure may or may not remain *elastic*. Therefore, depending upon the state of the structure, stiffness may be *non-linear-elastic* or *non-linear-inelastic*. The estimation of stiffness of the structure is required, both for static analysis and dynamic analysis. For a complex structure consisting of large number of similar or different types of

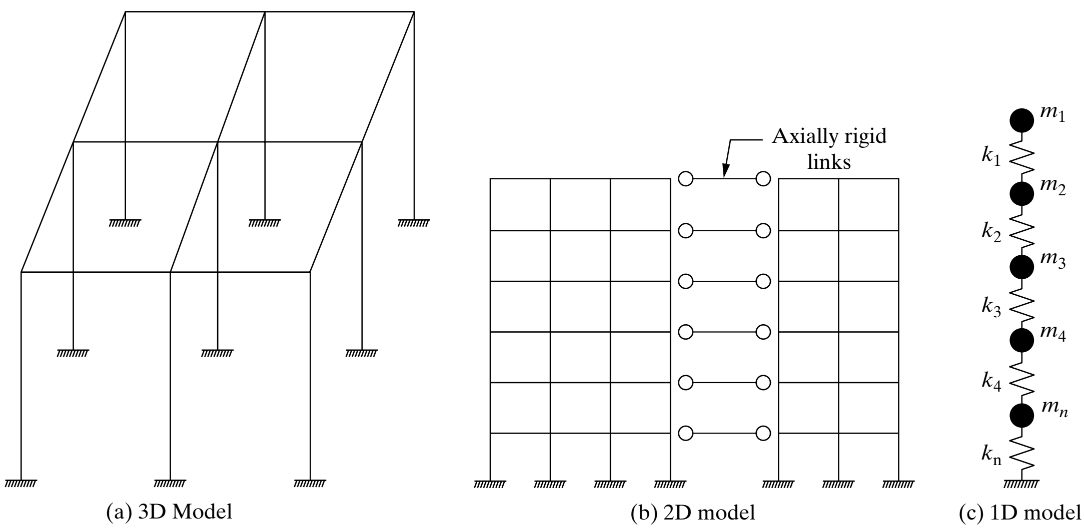


Figure 1.6 Modelling of a structure.

structural elements, stiffness is generated from the element level as done in the *matrix method of structural analysis*. The same procedure may be followed for dynamic analysis of structures. Sometimes it may not be easy to find solution of the mathematical model subjected to given loads in the absence of known appropriate boundary conditions. Therefore, it may be necessary to develop a numerical model of the problem and solve it using one or more *numerical techniques*.

Sometimes it is necessary to include soil and foundation in the model to examine the effect of soil–structure interaction. The problem now becomes very complex. The treatment requires advanced treatment that is beyond the scope of the present text.

Let us consider a simple pendulum consisting of a mass suspended from a massless rod as shown in Figure 1.7.

This pendulum can be displaced slightly horizontally and released gently. It sets into simple harmonic motion whose equation is given by the principles of physics, that is, *Newton's second law of motion*:

$$\ddot{x} \propto -x \quad (1.1a)$$

or,

$$\ddot{x} = -\omega^2 x \quad (1.1b)$$

where,

$$\ddot{x} = \frac{d^2x}{dt^2} \text{ acceleration} \quad (1.1c)$$

$$\omega = \sqrt{\frac{k}{m}} \quad (1.1d)$$

k = stiffness of the rod = AE/L
m = total mass
ω = natural frequency of vibration in rad/sec

$$\omega = \frac{2\pi}{T} = 2\pi f \quad (1.1e)$$

f = natural frequency of vibration in cycles/sec or in Hz

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \quad (1.1f)$$

T = natural period of vibration in sec

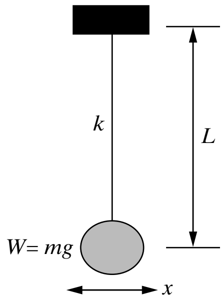


Figure 1.7 Simple pendulum.

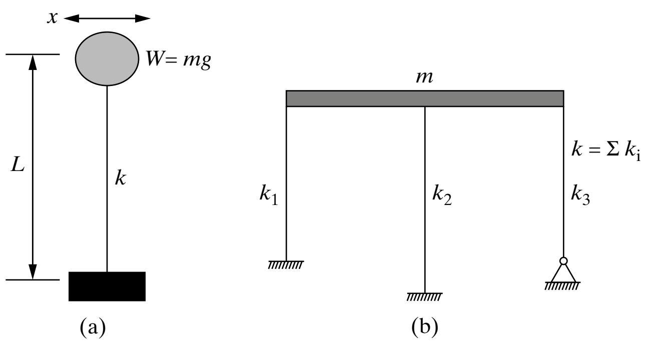


Figure 1.8 Inverted pendulum.

The pendulum has only one degree of freedom in the lateral direction, that is, lateral displacement x. It shows that the acceleration is directed towards the mean position of the pendulum and it is proportional to the displacement.

This pendulum can be inverted upside down as shown in Figure 1.8.

This inverted pendulum still vibrates in simple harmonic motion and its equation of motion remains same as before. In the inverted position, it can simulate the behaviour of an overhead water tank or that of a single-storey building. In case of a water tank, mass m represents the mass of the container and the water, whereas k represents the lateral stiffness of staging of the tank. In case of a building, mass m represents mass lumped at its roof, whereas k represents the lateral stiffness of all the lateral load-resisting elements. These elements may be columns, walls or both. The natural frequency of the pendulum is the most important characteristic of a dynamic system. It is a function of the *mass* and *stiffness* of the system.

Now let us consider a simple portal frame as shown in Figure 1.9(a). There are three degrees of freedom per node: one horizontal translation, one vertical translation and one rotation. Since A and D are fixed supports, the frame has six active degrees of freedom. If axial deformation is neglected, it will have three degrees of freedom: one lateral and two rotational for static analysis. In dynamic analysis, the number of independent displacements required to define the displaced position of all the masses relative to their original position is called *dynamic degrees of freedom*. If mass of the frame is concentrated at one location, that is roof, the frame will have only one degree

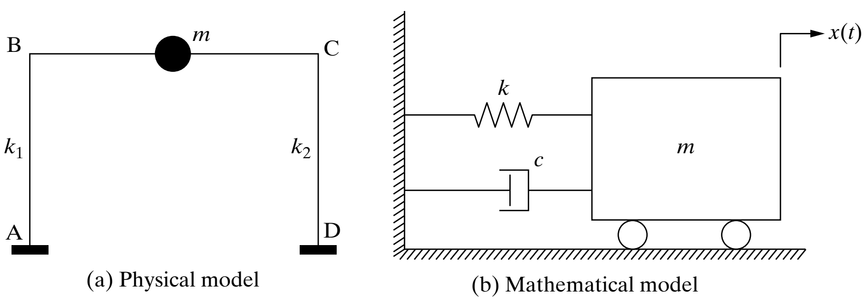


Figure 1.9 Portal frame.

of freedom—lateral displacement. Thus, it is called a SDOF system. Its mathematical model is shown in Figure 1.9(b). Again, natural frequency of the frame is its most important characteristic. It is a function of the mass and lateral stiffness of the system. The lateral stiffness of this rigid jointed frame depends upon the relative stiffness of beams and columns. Let us consider two extreme cases:

Case 1: *Rigid beam, that is, flexural stiffness of beam is infinite.*

The flexural rigidity of each column is EI. Height of each column is L. Therefore, lateral stiffness of each column is given by

$$K = \frac{12EI}{L^3} \tag{1.2a}$$

The total lateral stiffness of the frame =

$$K = 2 \times \frac{12EI}{L^3} \tag{1.2b}$$

Case 2: *Flexible beam, that is, flexural stiffness of beam is very low*

Therefore, lateral stiffness of each column is given by

$$K = \frac{3EI}{L^3} \tag{1.3a}$$

The total lateral stiffness of the frame =

$$K = 2 \times \frac{3EI}{L^3} \tag{1.3b}$$

In case there are three identical columns, the total stiffness of the frame becomes three times the stiffness of a single column. Equations (1.2a) and (1.3a) give range of lateral stiffness of a column.

1.3 DISCRETE AND CONTINUUM MODELLING

In dynamics of structures, the first step is to identify dynamic degrees of freedom. In certain structures these degrees of freedom are obvious and identifiable whereas, in many other structures it is difficult to identify these degrees of freedom. Thus, depending upon the number of degrees of freedom, we may have a multi-degrees-of-freedom system. For example, consider a four-storey moment-resistant frame having rigid floors. It is a skeletal structure and it is referred to as a discrete system in matrix structural analysis terminology. The entire mass of the floor and storey can be lumped at each floor level. It can, therefore, be identified that there is only one lateral displacement at each floor, that is, there is one-degree-of-freedom per floor. Similarly, in an overhead water tank, the entire mass of water is lumped at its top. The mass of water is much more than the mass of the staging. Thus, it can be said that there is only one lateral degree of freedom at the centre of gravity of the mass. This is referred to as a lumped mass approach in a discrete system. A TV tower or a truss steel bridge is an example of discrete structural system having n-degrees-of-freedom as shown in Figure 1.10.